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To the issue of modeling the pressure distribution in a gas-bearing formation

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Abstract

Based on the non-stationary Leibenzon piezoconductivity problem, there is considered the use of the finite-element difference method for estimating the pressure distribution in a gas-bearing formation. Based on the modeling results for a flat design scheme, it has been established that the process of gas-bearing formation depletion is mainly determined by the configuration of the location of production and injection wells, their capacities and filtration parameters. It has been shown that deconcentration of the system of production and injection wells is important for reducing reservoir depletion over time. This procedure is especially relevant for low-permeability formations. As the formation's permeability increases, the approach of the gas phase to production wells increases and the critical zone of depletion decreases. On the other hand, the increased permeability of a gas-bearing layer, during its operation without proper support, quickly leads to its exhaustion. In general, the process of depletion of a separate active section of a gas-bearing layer depends nonlinearly on its permeability. It has been shown that it is advisable to carry out increased production of raw materials at the initial stages of exploitation of a gas-bearing layer. As the formation is depleted, it is necessary to gradually reduce the production, which will lead to the prolongation of the exploitation of the formation. At the same time, it is important to maintain a technically justified value of the permeability of the working area of the formation.

Keywords: computer modeling, increasing of the reservoir's production period, factors of gas-bearing formations depletion.

Introduction

Currently, the problems of efficient operation of gas and gas condensate fields remain relevant [1–4]. These problems are primarily related to maintaining gas recovery from formations and achieving cost-effective methods of deposits exploitation.

In this situation, methods of computer modeling of productive formations are in demand, as they allow us to systematically monitor the situation around existing gas production wells and obtain the necessary information to effectively support stable gas production. They also make it possible to evaluate and take into account the filtration characteristics of the productive formation and uncertainties due to insufficient information about the structure and properties of the formation beyond the reach of existing wells. All this information can be obtained in a relatively cheap way and used for effective analysis, control and rational management of gas wells operation.

At the moment, there are many computer modeling methods used in various applied problems of gas production [4–8]: study of gas filtration processes under various technological actions on the reservoir; choosing an effective method of field development; maintaining rational well operation modes; assessment of residual reserves and stagnant zones in productive formations and optimal ways to use them; analysis and reduction of development risks and ensuring stable and long-term operation of a system of gas production wells.

On the other hand, a number of problems remain related to the accuracy and adequacy of modeling productive formations during the operation of gas production wells. In particular, the problem of effective regulation of filtration processes in wells (possibly with the participation of modern technologies [5–7]) for long-term and stable gas production remains relevant.

The use of the finite element-difference method [8, 10, 11] for solving the non-stationary problem of piezoelectric conductivity for the gas phase (Leibenzon problem), which takes into account the heterogeneity of the distribution of filtration parameters of the underlying layer and the infiltration parameters at its boundaries, provides a high solution to the problem. This makes it possible to adequately simulate the distribution of gas pressure under well operating conditions and determines some advantages over existing methods.

Formulation and method of solving the problem

In the future, we will consider productive gas-bearing formations in which the content of the liquid phase can be neglected. Assuming that the average thickness of the considered gas-bearing productive layer is much smaller than its horizontal dimensions, we will

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use the two-dimensional isotropic non-stationary model of piezoconductivity for the Leibenzon problem [4–9]. In this case, the general statement of the problem of piezoconductivity, taking into account the infiltration of the gas phase at the boundaries of the considered gas-bearing layer, in the Cartesian coordinate system (x, y) has the form [5–7]:

$$\frac{\partial P^2}{\partial t} = \chi(\frac{\partial^2 P^2}{\partial x^2} + \frac{\partial^2 P^2}{\partial y^2}) + \gamma; \quad (1)$$

$$P(t=0) = P_0; \quad (2)$$

$$k_g \, gradP^2 = \alpha_g (P^2 - P_b^2). \quad (3)$$

Here (1) is the Leibenzon piezoelectric conductivity equation; (2) is the initial condition; (3) is the limiting condition for gas phase infiltration within the considered area of the formation; $P(x,y,t)$ is the pressure as a function of coordinates and time; $\chi = \frac{kP_0}{\eta m}$ is the Leibenzon piezoelectric conductivity coefficient; k is the permeability of the gas phase of the formation; k_g is the permeability of the gas phase within the formation; η is the dynamic viscosity of gas; m is the porosity of the gas condensate formation; γ is the power parameter of the production or injection well; P_0 is the initial pressure in the formation; α_g is the gas phase infiltration coefficient within the considered area of the formation; P_b is he pressure at the boundaries of the reservoir area.

To solve the non-stationary problem of piezoelectric conductivity (1) – (3), a variational finite element method was used [8, 10, 11], leading to the solution of the variational equation of piezoelectric conductivity

$$\delta I(P) = 0. \quad (4)$$

Here $I(P)$ is the functional of the Leibenzon piezoconductivity problem (1) – (3), which, when replaced $\tilde{P} = P^2$, is represented in the usual form of the piezoconductivity problem [5]:

$$I(\tilde{P}) = \frac{1}{2}\iint_S \{k[(\frac{\partial \tilde{P}}{\partial x})^2 + (\frac{\partial \tilde{P}}{\partial y})^2] + 2\int_{P_0}^{P} \frac{k}{\chi}\frac{\partial \tilde{P}}{\partial t} d\tilde{P} - 2\gamma \tilde{P}\} dxdy - \frac{1}{2}\int_L \alpha_g (\tilde{P} - 2\tilde{P}_b)\tilde{P} dl; \quad (5)$$

S is the cross-sectional area of the studied area; L is the contour covering area S; *dl is the* contour element.

When solving the variational equation (4), an eight-node isoparametric quadrangular finite element has been used [8, 10, 11]. The Cartesian system (x, y) is used as a global coordinate system, where all finite elements into which the area S is divided are united. The normalized coordinate system (ξ, η) is used as a local coordinate system, where approximation functions φ_i are determined based on quadratic polynomials and numerical integration is carried out within the finished element [8, 10, 11]. Note that the use of an isoparametric finite element based on quadratic approximation leads to high convergence and stability of the solution to the problem. The finite element approximation functions are:

$$\begin{cases} \varphi_1 = \frac{1}{4}(1-\zeta)(1-\eta)(-\zeta-\eta-1), \\ \varphi_2 = \frac{1}{4}(1+\zeta)(1-\eta)(\zeta-\eta-1), \\ \varphi_3 = \frac{1}{4}(1+\zeta)(1+\eta)(\zeta+\eta-1), \\ \varphi_4 = \frac{1}{4}(1-\zeta)(1+\eta)(-\zeta+\eta-1), \\ \varphi_5 = \frac{1}{2}(1-\zeta^2)(1-\eta), \\ \varphi_6 = \frac{1}{2}(1-\eta^2)(1+\zeta), \\ \varphi_7 = \frac{1}{2}(1-\zeta^2)(1+\eta), \\ \varphi_8 = \frac{1}{2}(1-\eta^2)(1-\zeta). \end{cases} \quad (6)$$

In this system, the coordinates, pressure, initial reservoir pressure, pressure at the boundaries of the considered region of the reservoir, the gas phase infiltration coefficient at the boundaries of the considered region, as well as the derivatives of the pressure along the coordinates are approximated as follows

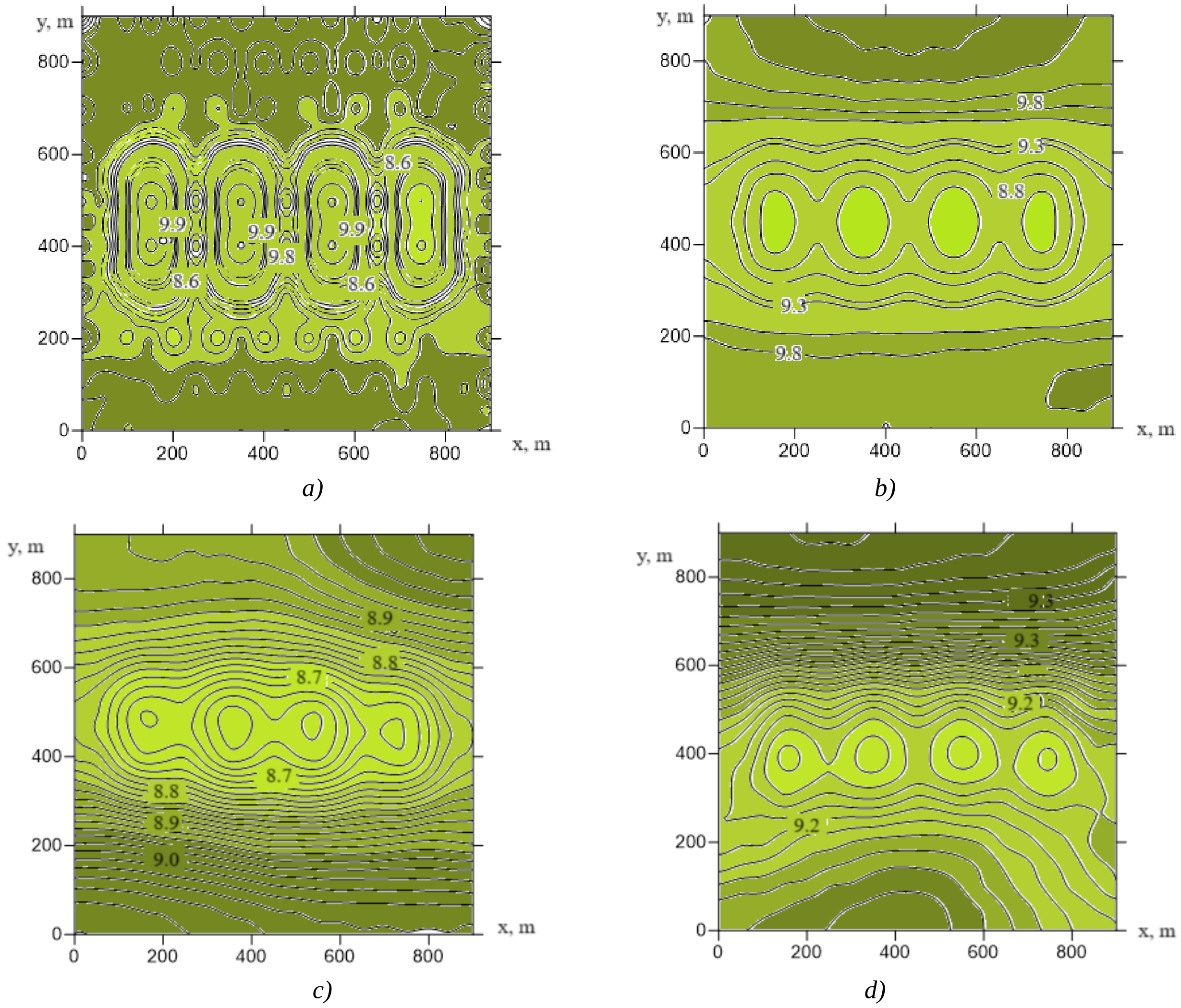
$$\begin{cases} x = \sum_{i=1}^{8} x_i \varphi_i, \; y = \sum_{i=1}^{8} y_i \varphi_i, \\ \tilde{P} = \sum_{i=1}^{8} P_i \varphi_i, \; \tilde{P}_0 = \sum_{i=1}^{8} P_{0i} \varphi_i, \\ \tilde{P}_г = \sum_{i=1}^{8} P_{гi} \varphi_i, \; \alpha^2 = \sum_{i=1}^{8} \alpha_i \varphi_i, \\ \frac{\partial \tilde{P}}{\partial x} = \sum_{i=1}^{8} P_i \Psi_i, \; \frac{\partial \tilde{P}}{\partial y} = \sum_{i=1}^{8} P_i \Phi_i, \\ \Psi_i = \frac{1}{|J|}(\frac{\partial \varphi_i}{\partial \eta}\frac{\partial y}{\partial \xi} - \frac{\partial \varphi_i}{\partial \xi}\frac{\partial y}{\partial \eta}), \\ \Phi_i = \frac{1}{|J|}(\frac{\partial \varphi_i}{\partial \xi}\frac{\partial x}{\partial \eta} - \frac{\partial \varphi_i}{\partial \eta}\frac{\partial x}{\partial \xi}), \end{cases} \quad (7)$$

where $J = \frac{\partial y}{\partial \xi}\frac{\partial x}{\partial \eta} - \frac{\partial y}{\partial \eta}\frac{\partial x}{\partial \xi}$ is the Jacobian of the transition between coordinate systems (x, y) and (ξ, η).

Based on the variational equation (4) and assuming that the nodal values of the time derivatives of pressure $\frac{dP_i}{dt}$ are known values and do not vary, we will formulate a system of differential equations for the n-th node of the p-th finite element in the form

$$\frac{\partial I_p}{\partial P_n} = \sum_{i=1}^{8} \{H_{ni}^p \frac{dP_i}{dt} + (A_{ni}^p + Q_{ni}^p)P_i - Q_{ni}^p P_0^i\} - \gamma_n^p = 0, \quad (8)$$

where



a) k = 1.2 mD; b) k = 12 mD; c) k = 120 mD; d) k = 1200 mD
(thin crosses indicate the location of production wells)

Figure 1 – Pressure distribution around a concentrated system of gas production wells with the filtration parameters specified above and various formation permeability coefficients after 30 days of continuous operation

$$H_{ij}^{p} = \int_{-1}^{1}\int_{-1}^{1} \frac{k^{p}}{\chi^{p}} \varphi_i \varphi_j \left|J\right| d\xi d\eta,\; A_{ij}^{p} =$$

$$= \int_{-1}^{1}\int_{-1}^{1} k^{p} (\Psi_i \Psi_j + \Phi_i \Phi_j) \left|J\right| d\xi d\eta,$$

$$Q_{ij}^{p} = \int_{L} \alpha_j^{p} \varphi_i \varphi_j dl,\; \gamma_i^{p} = \int_{-1}^{1}\int_{-1}^{1} \gamma^{p} \varphi_i \left|J\right| d\xi d\eta.$$

To solve the system of linear differential equations of the first order (8) under initial conditions selected from the set of approximation formulas (7), the finite difference method is used, in which the approximation of the time derivative is carried out on the basis of an implicit difference scheme

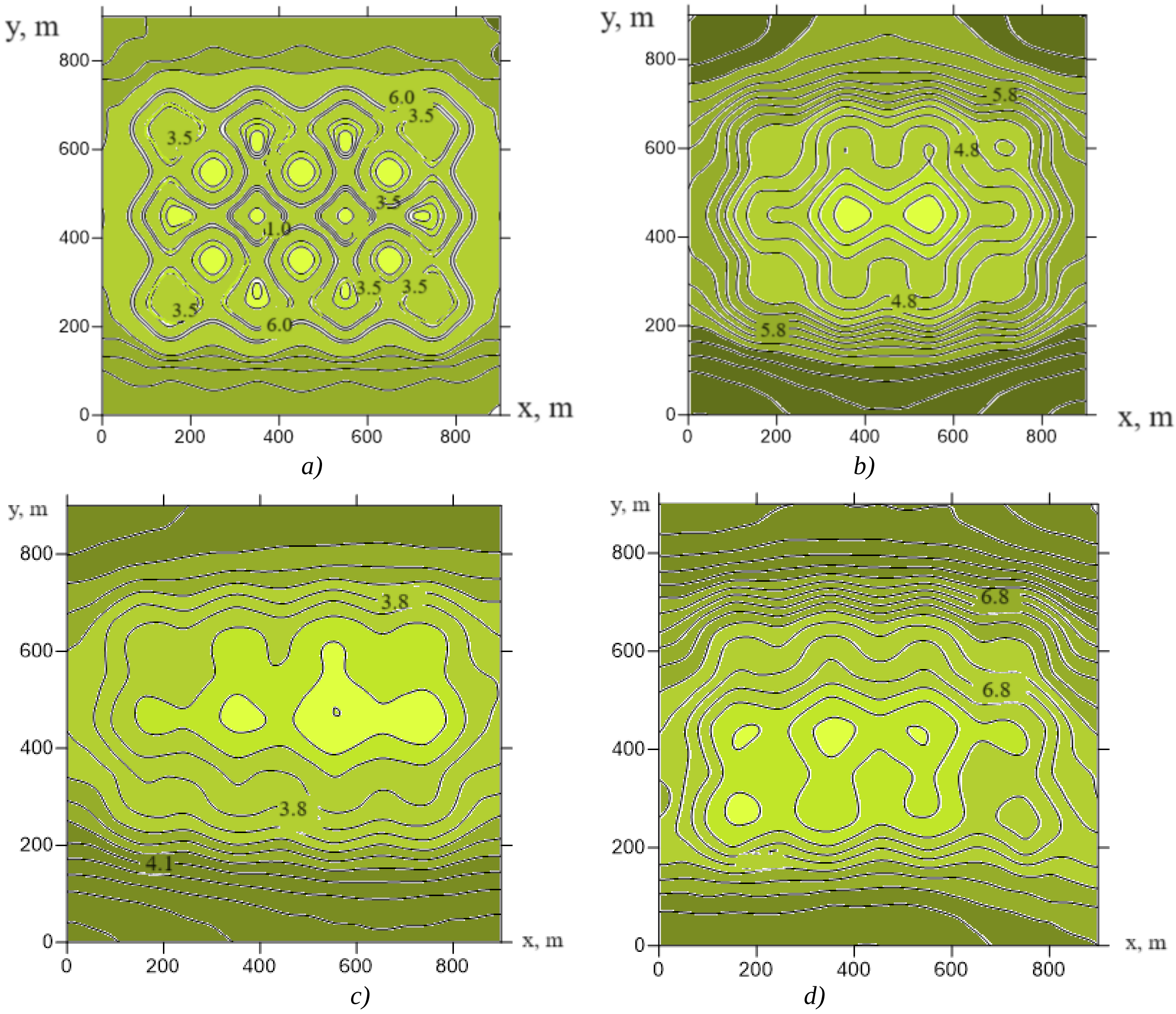
$$\frac{d\tilde{P}}{dt} = \frac{\tilde{P}(t+\Delta t) - \tilde{P}(t)}{\Delta t}. \qquad (9)$$

Substituting expression (9) into system (8), we obtain the following system of linear algebraic equations:

$$\sum_{i=1}^{8} \{ (\frac{1}{\Delta t} H_{ni}^{p} + A_{ni}^{p} + Q_{ni}^{p}) P_i (t+\Delta t) - \frac{1}{\Delta t} H_{ni}^{p} P_i(t) - Q_{ni}^{p} P_0^{i} \} -$$

$$-\gamma_n^{p} = 0, \quad n = \overline{1,8}. \qquad (10)$$

By summing equations (9) over all finite elements, we obtain a global system of linear algebraic equations that allows us to determine unknown pressure values at an instant in time $t + \Delta t$ through their value at the previous instant in time t. The solution of the global system of equations is carried out on the basis of the numerical Gaussian method without selecting the main element [8, 10]. As a result of the solution, the pressure is determined at all nodal points of the finite element mesh. Based on the found nodal values, the pressure is determined at an arbitrary point in the gas-bearing formation under consideration at a given point in time.



a) k = 1.2 mD; b) k = 12 mD; c) k = 120 mD; d) k = 1200 mD

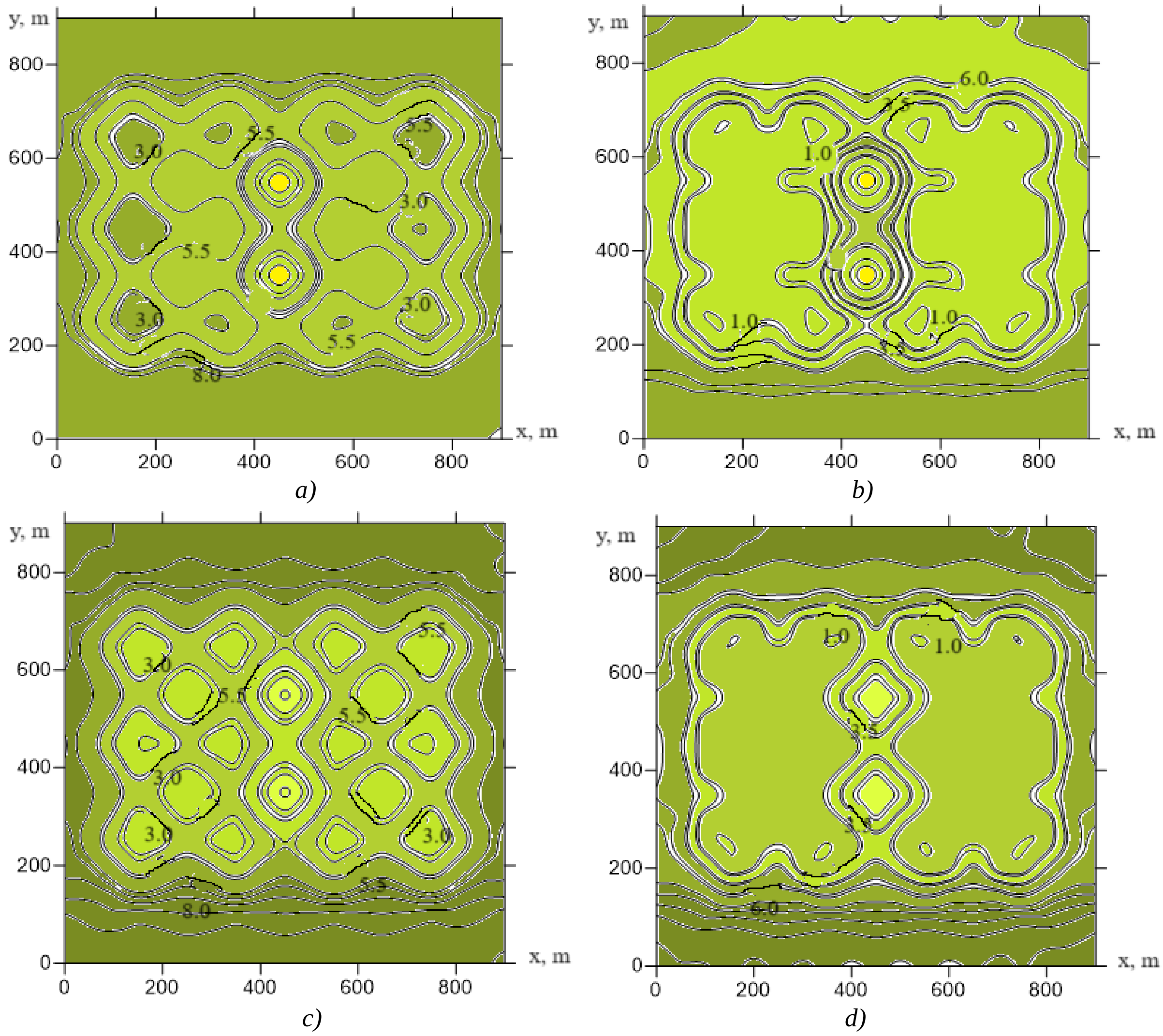
Figure 2 – Pressure distributions around a dispersed system of gas production wells with the filtration parameters specified above and various formation permeability coefficients after 150 days of continuous operation

Analysis of some results

Let's consider the processes of filtration of a productive gas-bearing formation around 12 production wells with a capacity of 24840 m^3 per day. We suggest that the dimensions of the gas-bearing formation under consideration are 900×900 m^2. Let us choose some characteristic average parameters of the gas-bearing formation [5]: k = 1.2 mD = $0.12 \cdot 10^{-14}$ m^2; m=0.15; η = $0.18 \cdot 10^{-4}$ Pa·s; $P_0 = 10$ MPa. In this case, the piezoelectric conductivity coefficient of leu-benzone is $\chi = 0.45 \cdot 10^{-2}$ m^2/s. When modeling, we will consider the boundaries of the considered reservoir area to be impermeable, that is, the gas phase infiltration coefficient within the area is zero $\alpha_g = 0$.

Figure 1, *a–d* show the pressure distribution around a concentrated system of gas production wells (three strips of 4 wells, at a distance of 100 m between them, located nearby, the distance between the strips is also 100 m) at different formation permeability coefficients after 30 days of continuous action. Figure 1 illustrates the depletion of a gas-bearing formation under given operating conditions over a short period of time (30 days). We see that the greatest depletion of the considered region of the formation occurs in the case when this region has minimal permeability (Fig. 1, *a*). As the permeability of the formation increases due to an increase in the influx of the gas phase, the depletion process decreases and the pressure distribution becomes more uniform throughout the entire formation area. Locked contours in this case show localized areas of pressure distribution around active wells and make it possible to determine the depletion state of these areas.

Figure 2, *a–d* show cases of pressure distribution around a concentrated system of active wells (the three strips discussed above are located in parallel at a distance of 200 m from one another) at different formation permeability coefficients after 150 days of continuous operation. We see that due to the dispersal of extracted wells, the influx of gas phase to the wells



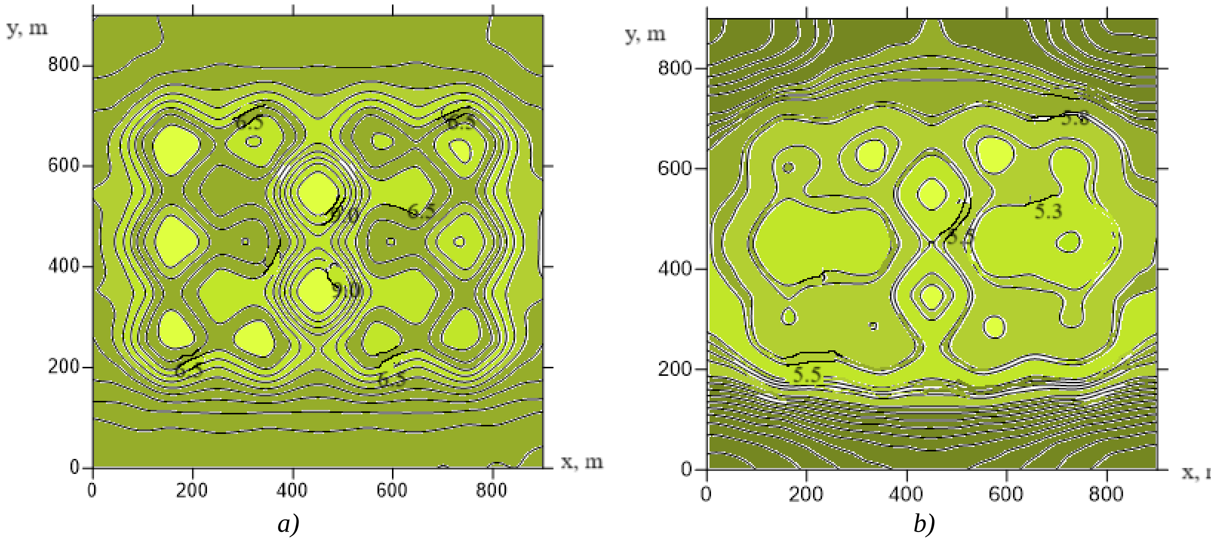
a) after 150 days; b) after 250 days; c) with twice reduced capacities of injection wells after 150 days; d) with twice reduced capacity of injection wells after 250 days (bold crosses indicate the location of injection wells)

Figure 3 – Pressure distributions around a dispersed system of gas production wells with the addition of two injection wells, with the above parameters of reservoir production

increases and the critical depletion area of the working section of the formation is significantly reduced, which allows its operation to continue over time. On the other hand, the analysis (Fig. 2, *c*) shows that in this case the process of reservoir depletion nonlinearly depends on its permeability. Thus, in each practical case, it is necessary to carry out a set of experimental procedures to determine and maintain the optimal operating value of the permeability of the gas-bearing section of the formation in operation.

Figure 3, *a* show cases of pressure distribution around a dispersed system of production wells with the addition of two injection wells of the same capacity (24840 m^3 of gas per day) within the considered reservoir area after 150 and 250 days, respectively. Figure 3, *c*, *d* show the previous cases, but with a twice reduced capacity of injection wells. We see that if after 150 days, due to the injection actions of two wells, the state of reservoir depletion remains at a satisfactory level (Fig. 3, *a*) then after 250 days (Fig. 3, *b*) it reaches a critical level. The same applies to the cases of Figures 3, *c* and 3, *d*. At the same time, comparing Figures 3, *a* and 3, *c* we see approximately the same pressure distribution, regardless of the capacity of the injection wells. This allows us to conclude that in the early stages of operation it is advisable to carry out more intensive exploitation of the gas-bearing formation.

Figure 4, *a*, *b* show cases of pressure distribution around a dispersed system of production wells with twice reduced capacity (12420 m^3 of gas per day) with the addition of two injection wells of the same reduced capacity inside the formation at different values of formation permeability after 250 days. In both cases, we see a satisfactory state of depletion of the gas-bearing formation over a fairly long period of operation. Thus, we can conclude that in the later stages of exploitation of a gas-bearing formation, it makes sense to gradually reduce the level of production in order to extend the full exploitation of this formation section.



a) k = 1.2 mD; b) k = 12 mD

Figure 4 – Pressure distributions around a dispersed system of gas production wells with the addition of two injection wells, with twice reduced capacities of production and injection wells after 250 days

Conclusions

A finite element-difference method has been used to solve the non-stationary Leibenzon piezoelectric conductivity problem to assess the pressure distribution in productive gas-bearing formations. The modeling results for a flat design scheme show that the depletion process is mainly determined by the configuration of the location of production wells, their capacities and filtration parameters of the productive formation (especially permeability).

It has been established that the distribution of production wells plays an important role in reducing the process of reservoir depletion over time. This procedure is especially important for formations with low permeability. As formation permeability increases, the approach of the gas phase to production wells increases and the critical depletion zone decreases. On the other hand, the increased permeability of a section of a gas-bearing formation, during its operation without proper support, quickly leads to its depletion. In general, the process of depletion of a separate active section of a gas-bearing formation may nonlinearly depend on its permeability.

It is shown that at the initial stages of exploitation of a gas-bearing formation site, it is advisable to carry out intensive extraction of raw materials. With the depletion of this section of the formation, it is necessary to gradually reduce production, which will lead to an extension of the operation of the formation. To maintain a given level of gas production, it is necessary to use optimally located injection wells. It is shown that over time, the need to maintain the formation using injection wells increases.

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До питання моделювання розподілу тиску в газоносному пласті

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На основі нестаціонарної задачі п'єзопровідності Лейбензона розглянуто використання скінченно-елементно-різницевого методу для оцінки розподілу тиску в газоносному пласті. За результатами моделювання для плоскої розрахункової схеми встановлено, що процес виснаження газоносного пласта в основному визначається конфігурацією розташування видобувних і нагнітальних свердловин, їх дебітом та параметрами фільтрації. Було показано, що деконцентрація системи видобувних і нагнітальних свердловин є важливою для зменшення виснаження колектора з часом. Особливо ця процедура актуальна для малопроникних утворень. З підвищенням проникності пласта збільшується наближення газової фази до видобувних свердловин і зменшується критична зона виснаження. З іншого боку, підвищена проникність газоносного пласта при його експлуатації швидко призводить до його виснаження. Загалом, процес виснаження окремої активної ділянки газоносного пласта нелінійно залежить від його проникності. Показано доцільність збільшення видобутку сировини на початкових етапах експлуатації газоносного пласта. У міру виснаження пласта необхідно поступово зменшувати видобуток, що призведе до подовження термінів експлуатації. При цьому важливо підтримувати технічно обґрунтоване значення проникності робочої зони пласта.

Ключові слова: *збільшення продуктивності пласта, комп'ютерне моделювання, фактори виснаження газоносних пластів.*